Superconductivity

The basic facts:

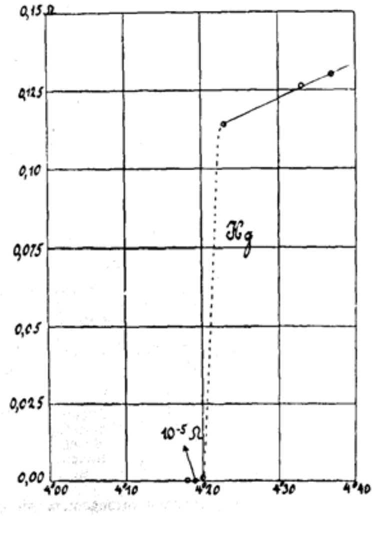
- Resistivity goes to zero below the critical temperature T_c (the most sensitive measurements imply $R < 10^{-25}\ \Omega$)
- Many different materials show superconductivity
- T_c values range from a few mK up to 160K
- Superconductors expel flux (the **Meissner effect**) and act as perfect diamagnets.
- Superconductivity is destroyed by a critical magnetic field B_c
- Specific heat, infrared absorption, tunnelling, .. all imply that there is an energy gap associated with superconductivity

Resistivity

Transition is very sharp in pure materials (as narrow as 10^{-3} K), broader when impurities are present.

Very good conductors (simple free electron materials) do not superconduct.

Superconductivity is destroyed by high currents (critical current J_c)



Superconducting Elements

Table 1 Superconductivity parameters of the elements

An asterisk denotes an element superconducting only in thin films or under high pressure in a crystal modification not normally stable. Data courtesy of B. T. Matthias, revised by T. Geballe.

Transition temperature in K
Critical magnetic field at absolute zero in gauss (10^{-4} tesla)

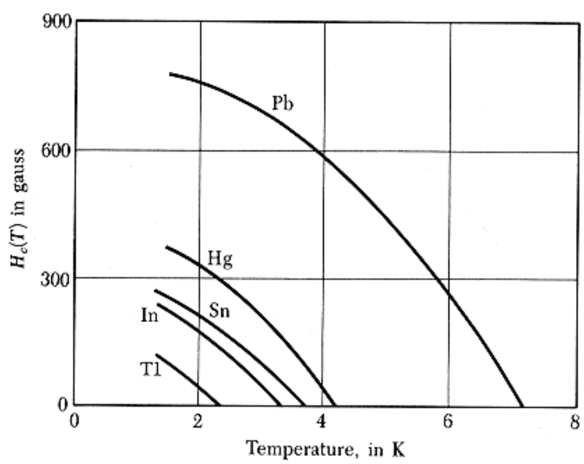
Li	Be 0.026											B	C	N	O	F	Ne
Na	Mg											Al 1.140 105	Si*	P*	S*	Cl	Ar
K	Ca	Sc	Ti 0.39 100	V 5.38 1420	Cr*	Mn	Fe	Co	Ni	Cu	Zn 0.875 53	Ga 1.091 51	Ge*	As*	Se*	Br	Kr
Rb	Sr	Y*	Zr 0.546 47	Nb 9.50 1980	Mo 0.92 95	Tc 7.77 1410	Ru 0.51 70	Rh .0003 .049	Pd	Ag	Cd 0.56 30	In 3.4035 293	Sn (w) 3.722 309	Sb*	Te*	I	Xe
Cs*	Ba*	La fcc 6.00 1100	Hf 0.12	Ta 4.483 830	W 0.012 1.07	Re 1.4 198	Os 0.655 65	Ir 0.14 19	Pt	Au	Hg (α) 4.153 412	Tl 2.39 171	Pb 7.193 803	Bi*	Po	At	Rn
Fr	Ra	Ac															
			Ce*	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu 0.1	
			Th 1.368 1.62	Pa 1.4	U*(α)	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr	

Critical Field

Superconductivity is destroyed by magnetic fields

Critical field depends on temperature, typically

$$B_c = B_0 \, (1 - (T/T_c)^2)$$



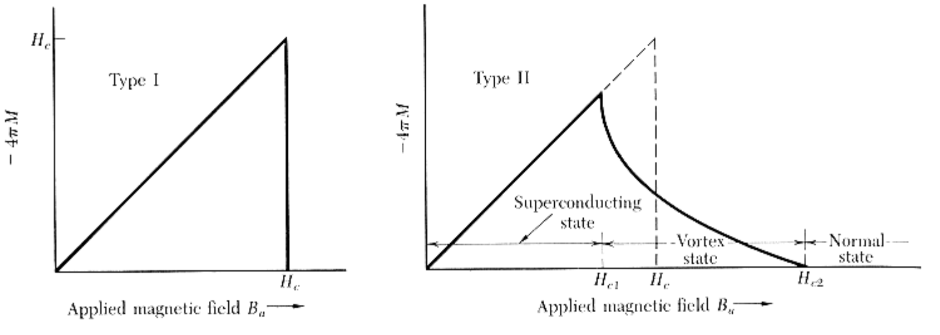
Meissner Effect

It was discovered in 1933 that when cooled in a magnetic field flux is expelled completely from a superconductor

Inside the superconductor

$B = B_a + \mu_0 M$, giving $M = -B/\mu_0$ ($\chi = -1$)

This is **not** the result of zero resistance

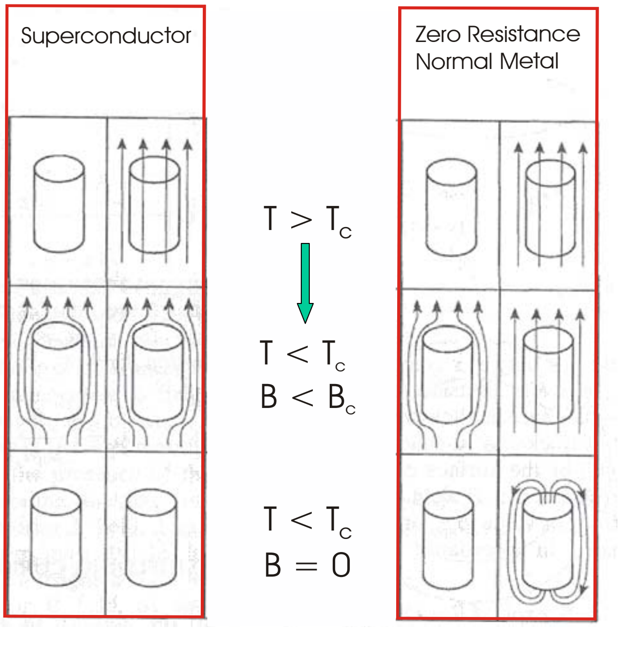


Flux expulsion

$$\oint \mathbf{E}.d\mathbf{s} = \partial\phi/\partial t$$

Flux is expelled from superconductor,

Flux is trapped in a zero resistance metal



Flux penetration

In order to cause the flux expulsion it is necessary for there to be a surface current to generate the internal flux

$$\mathbf{j} = \frac{ne^2\tau}{m}\mathbf{E}\ ,\ so\ \frac{\partial \mathbf{j}}{\partial t} = \frac{ne^2}{m}\mathbf{E}$$

$$curl\frac{\partial \mathbf{j}}{\partial t} = \frac{ne^2}{m}curl\mathbf{E} = -\frac{ne^2}{m}\frac{\partial \mathbf{B}}{\partial t}$$

London & London **assumed** that: $curl\ \mathbf{j} = -\frac{ne^2}{m}\mathbf{B}$

$$curl\ curl\ \mathbf{j} = -\frac{ne^2}{m}curl\ \mathbf{B} = -\mu_0\frac{ne^2}{m}\mathbf{j} = -\nabla^2\mathbf{j}$$

$$\text{giving}:\ j = j_0\exp -\lambda x,\quad \text{with}\ \lambda = \sqrt{\frac{m}{\mu_0 ne^2}}$$

λ is the London penetration depth (approx. 10 nm)

Thermodynamics of the Superconducting phase transition

In magnetic field we define a Gibbs free energy as:

G = E - TS -**M.B**, where the **M.B** term includes the energy of interaction of the specimen with the external field. Thus:

dG = (dE - TdS - **B.**d**M**) - SdT - **M**.d**B** = - SdT - **M**.d**B**

dE = dQ + dW

$$G_S(B_c,T) = G_S(0,T) - \int_0^{B_c}\mathbf{M}.d\mathbf{B}\ ,\ \text{with}\ \mathbf{M} = -\frac{\mathbf{B}}{\mu_0}$$

$$G_S(B_c,T) = G_S(0,T) + \int_0^{B_c}\frac{B}{\mu_0}dB = G_S(0,T) + \frac{B_c^2}{2\mu_0}$$

At B_c the normal and superconducting phases are in equilibrium, so their Gibbs functions are the same. Thus:

$$G_N(0,T) - G_S(0,T) = \frac{B_c^2}{2\mu_0}$$

We can deduce the Entropy difference from S = -∂G/∂T

$$\Delta S = S_N - S_S = -\frac{1}{2\mu_0}\frac{dB_c^2}{dT} = -\frac{B_c}{\mu_0}\frac{dB_c}{dT}$$

At T_c the value of $B_c \rightarrow 0$ so $S_N = S_S$

dB_c/dT is negative, so $S_N > S_S$ for $T < T_c$

Entropy and Specific Heat

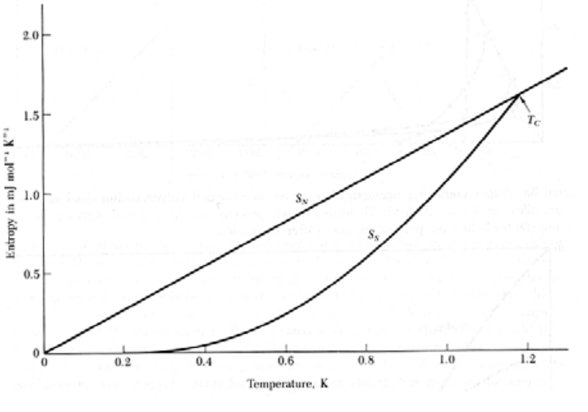
Entropy of two states is the same at T_c

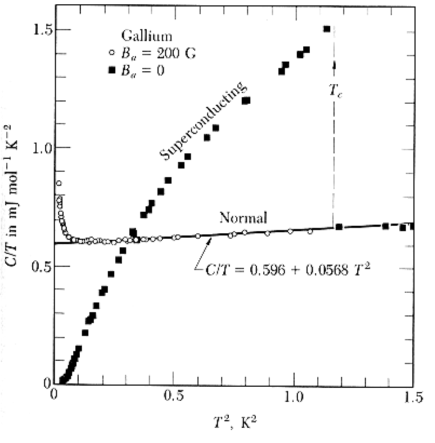
Specific heat is:

$$C = T\frac{\partial S}{\partial T}$$

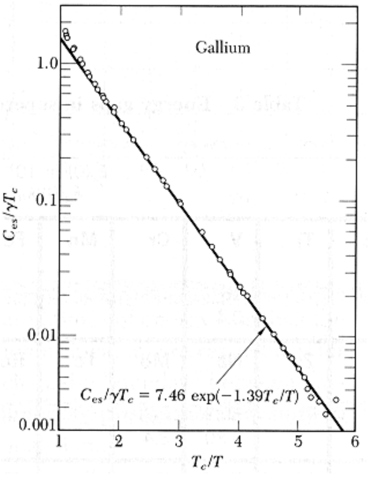
Discontinuity in C at T_c

Second order phase transition





Specific heat of superconductor has a large discontinuity and tends to zero at T = 0



Specific heat is activated with

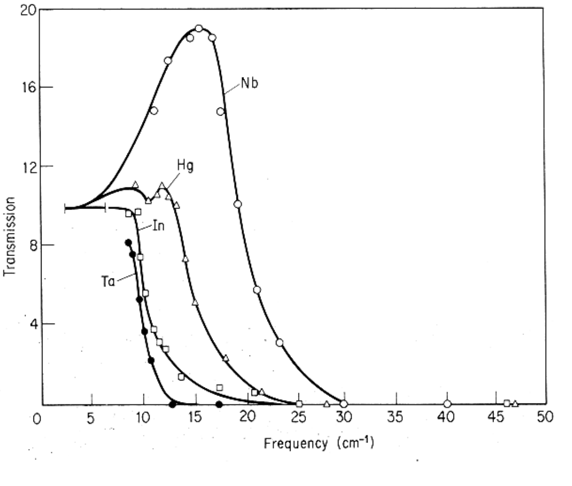
$$C_S \propto \exp\left(-\frac{\Delta}{k_B T}\right)$$

Infrared absorption

Infrared absorption when hν > 2Δ

Value of energy gap 2Δ is related to T_c

$$2\Delta \approx 3.5\, k_B T_c$$



BCS Theory

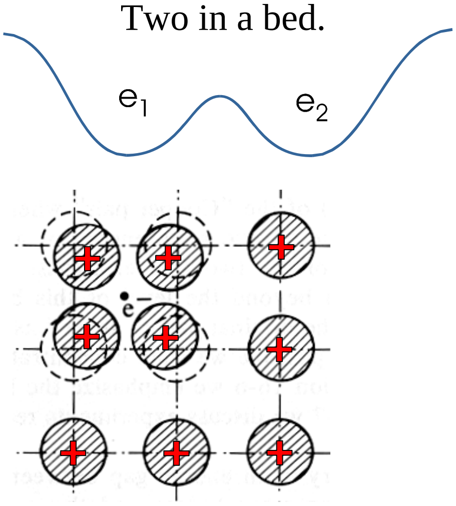
- A field theory developed by Bardeen, Cooper and Schrieffer
- Explanation for the formation of an energy gap
- based on the formation of 'Cooper pairs' of electrons
- electrons experience an attraction caused by interaction with crystal lattice leading to binding in pairs

Evidence for phonon interactions:

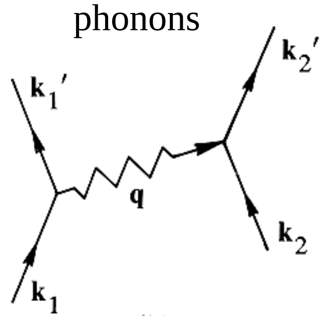
- Isotope effect. For different isotopes $T_c \propto M^{-1/2}$
- Good conductors at high temp. (Cu, Na, Au etc) do not superconduct, poor conductors do (Hg, Pb, Sn…)

Cooper pairs

Two in a bed.



Exchange of virtual phonons



Strongest interaction for $k_1 = -k_2$

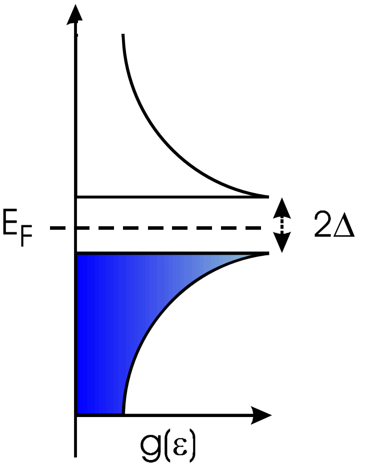
Electrons bind together in pairs with momenta k_F and $-k_F$. Bonding pair have opposite spins in a spin singlet wavefunction

$$\phi = \phi_S(r_1, r_2)\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$

Pair has charge 2e and mass 2m

Pairs gain a binding energy of Δ per electron

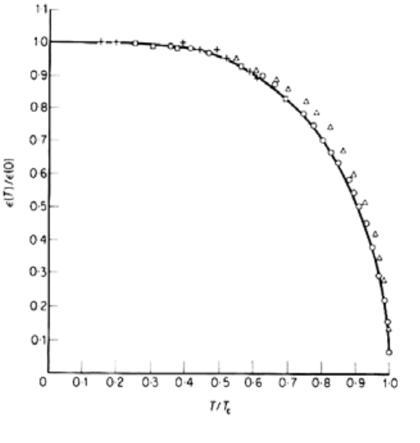
Energy gap of 2Δ occurs at the Fermi energy E_F



Zero Resistance

Current flows by displacement of entire Fermi surface. Because of the energy gap no scattering can occur until pairs can be excited across gap. Causes a Critical current J_c once electrons gain enough energy.

Energy gap is temperature dependent, leading to temperature dependence of B_c, J_c.



Energy and Coherence

Average energy gain per electron is approx. $\Delta/2$ (actually $\Delta/4$ with full theory) so as $\Delta \times g(E_F)$ electrons are shifted down

$$\frac{\Delta^2\, g(E_F)}{4} = \text{gain in Gibbs free energy} = \frac{B_c^2}{2\mu_0}$$

Coherence Length $\xi = v_F \tau$

Estimate τ from energy gap: $\hbar/\tau = 2\Delta$

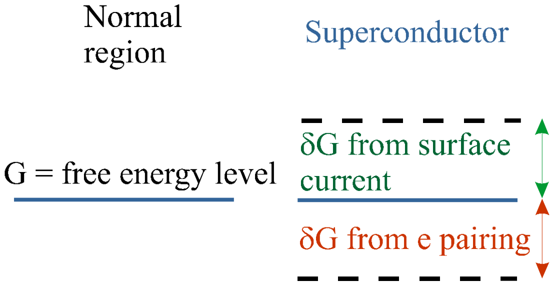
so $\xi = v_F\, \hbar/2\Delta$ (accurate result: $v_F\, \hbar/\pi\Delta$)

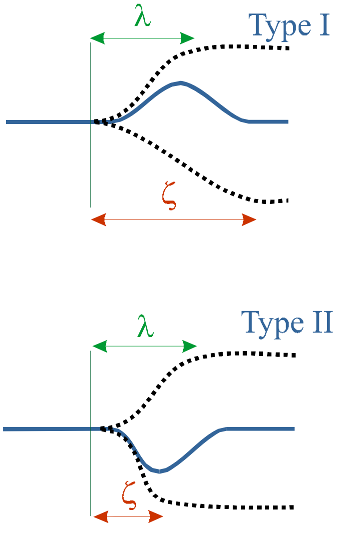
typical values are 1000 - 1 nm

(much shorter in *exotic* and high T_c materials)

Type I and Type II Superconductors in B field

Magnetisation energy occurs over the London penetration depth λ





Superconductivity is established over the coherence Length ξ

Type II superconductors

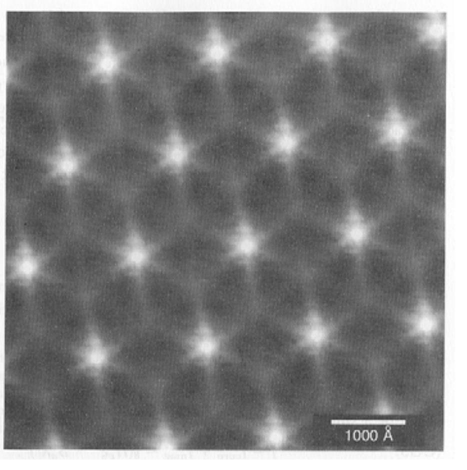
Have short coherence lengths and high T_c

Form vortices which make a flux lattice above B_c^1

Typical materials:

One element, Nb, and many alloys such as

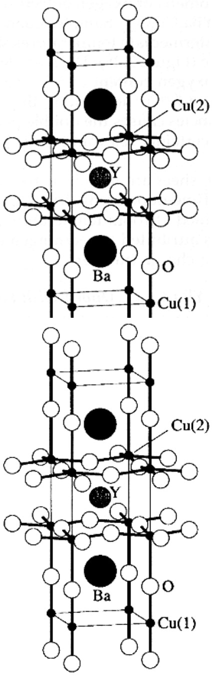
NbTi, Nb_3Sn, V_3Ga….



High T_c Materials:

$Ba_{0.75}La_{4.25}Cu_5O_{5(3-y)}$

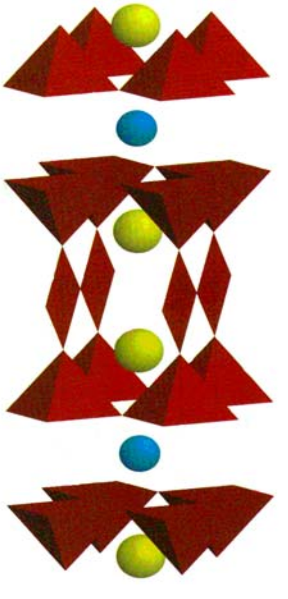
$YBa_2Cu_3O_{7-x}$



High T_c Materials

Conduction takes place in CuO planes

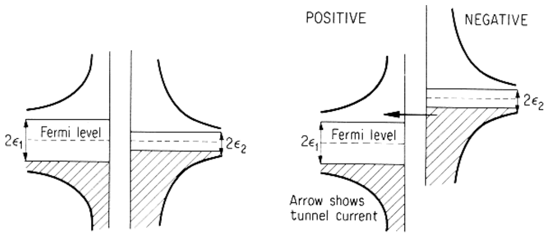
All properties are highly anisotropic

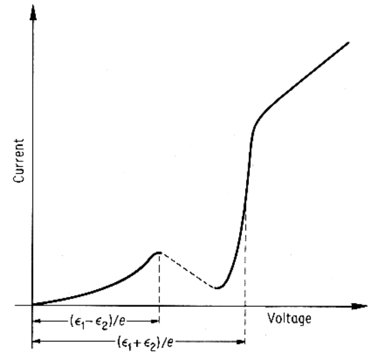


Superconducting Tunnelling

Tunnelling between two superconductors with a very thin (few nm) barrier

Tunnel current shows features due to alignment of energy levels either side of barrier. Measures energy gaps





Flux Quantisation

Resistivity = 0 means no scattering. Therefore there is macroscopic phase coherence of the supercurrent over the entire length of a superconductor

$$\mathbf{v} = \frac{1}{m}\left(-\hbar\nabla - q\mathbf{A}\right), \qquad \mathbf{j} = \frac{nq}{m}\left(-\hbar\nabla - q\mathbf{A}\right)$$

$$\oint \mathbf{j}.d\mathbf{s} = \frac{nq}{m}\left(\oint \hbar\,\mathbf{k}.d\mathbf{s} - \oint q\mathbf{A}.d\mathbf{s}\right)$$

Choose a path inside superconductor ⇨

$$0 = \frac{nq}{m}\left(2n\pi\hbar - q\oint curl\mathbf{A}.d\mathbf{S}\right)$$

$$\therefore\ 2n\pi\hbar = q\oint curl\mathbf{A}.d\mathbf{S} = q\oint \mathbf{B}.d\mathbf{S} = q\Phi$$

Result is that flux is quantised in units: $\Phi_0 = \frac{h}{q} = \frac{h}{2e} = 2.07\times10^{-15}\ T\ m^2$

Proof of existence of Cooper pairs.

Leads to many more sophisticated quantum interference effects (Josephson effect), and applications such as very sensitive measurement of small magnetic fields (and fluxes) e.g. SQUIDs